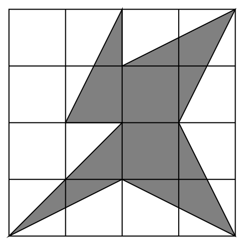
Time limit: 90 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. What is the sum of all two-digit odd numbers whose digits are all greater than 6?

2. Define an operation $\diamondsuit$ as $a \diamondsuit b = 12a - 10b$. Compute the value of $((((20 \diamondsuit 22) \diamondsuit 22) \diamondsuit 22) \diamondsuit 22)$.

3. For lunch, Lamy, Botan, Nene, and Polka each choose one of three options: a hot dog, a slice of pizza, or a hamburger. Lamy and Botan choose different items, and Nene and Polka choose the same item. In how many ways could they choose their items?

4. Big Chungus has been thinking of a new symbol for BMT, and the drawing below is what he came up with. If each of the 16 small squares in the grid are unit squares, what is the area of the shaded region?



5. Compute the last digit of $(5^{20} + 2)^3$.

6. To fold a paper airplane, Austin starts with a square paper $FOLD$ with side length 2. First, he folds corners L and D to the square's center. Then, he folds corner F to corner O. What is the longest distance between two corners of the *resulting* figure?

7. Let $f(x) = x^2 + \lfloor x \rfloor^2 - 2x \lfloor x \rfloor + 1$. Compute $f(4 + \frac{5}{6})$. (Here, $\lfloor m \rfloor$ is defined as the greatest integer less than or equal to m. For example, $\lfloor 3 \rfloor = 3$ and $\lfloor -4.25 \rfloor = -5$.)

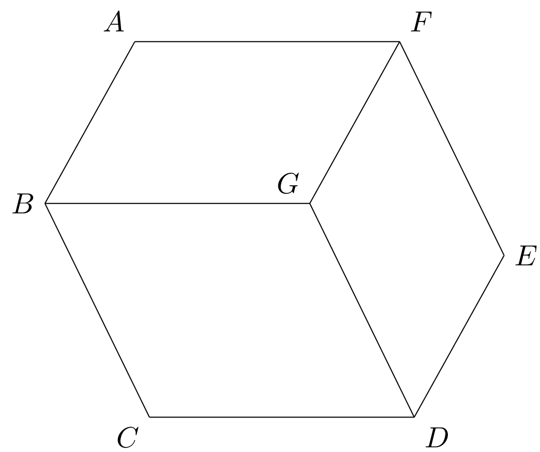
8. Oliver is at a carnival. He is offered to play a game where he rolls a fair dice and receives \$1 if his roll is a 1 or 2, receives \$2 if his roll is a 3 or 4, and receives \$3 if his roll is a 5 or 6. Oliver plays the game repeatedly until he has received a total of at least \$2. What is the probability that he ends with \$3?

9. What is the measure of the largest convex angle formed by the hour and minute hands of a clock between 1:45 PM and 2:40 PM, in degrees? Convex angles always have a measure of less than 180 degrees.

10. Each box in the equation
$$\square \times \square \times \square - \square \times \square \times \square = 9$$
is filled in with a different number in the list 2, 3, 4, 5, 6, 7, 8 so that the equation is true. Which number in the list is not used to fill in a box?

11. The equation
$$4^x - 5 \cdot 2^{x+1} + 16 = 0$$
has two integer solutions for x. Find their sum.

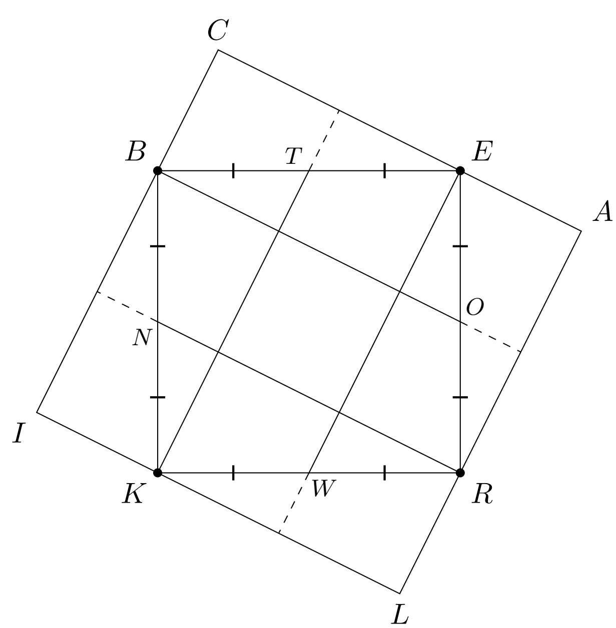
12. Parallelograms $ABGF$, $CDGB$ and $EFGD$ are drawn so that $ABCDEF$ is a convex hexagon, as shown. If $\angle ABG = 53^\circ$ and $\angle CDG = 56^\circ$, what is the measure of $\angle EFG$, in degrees?



13. Three standard six-sided dice are rolled. What is the probability that the product of the values on the top faces of the three dice is a perfect cube?

14. Compute the number of positive integer divisors of 100000 which do not contain the digit 0.

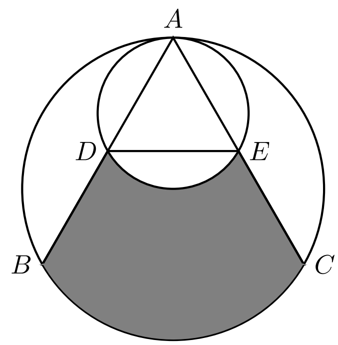
15. Sohom constructs a square $BERK$ of side length 10. Darlnim adds points T, O, W, and N, which are the midpoints of $\overline{BE}$, $\overline{ER}$, $\overline{RK}$, and $\overline{KB}$, respectively. Lastly, Sylvia constructs square $CALI$ whose edges contain the vertices of $BERK$, such that $\overline{CA}$ is parallel to $\overline{BO}$. Compute the area of $CALI$.



16. A street on Stanford can be modeled by a number line. Four Stanford students, located at positions 1, 9, 25 and 49 along the line, want to take an UberXL to Berkeley, but are not sure where to meet the driver. Find the smallest possible total distance walked by the students to a single position on the street. (For example, if they were to meet at position 46, then the total distance walked by the students would be $45 + 37 + 21 + 3 = 106$, where the distances walked by the students at positions 1, 9, 25 and 49 are summed in that order.)

17. Midori and Momoi are arguing over chores. Each of 5 chores may either be done by Midori, done by Momoi, or put off for tomorrow. Today, each of them must complete at least one chore, and more than half of the chores must be completed. How many ways can they assign chores for today? (The order in which chores are completed does not matter.)

18. Let equilateral triangle $\triangle ABC$ be inscribed in a circle ω_1 with radius 4. Consider another circle ω_2 with radius 2 internally tangent to ω_1 at A. Let ω_2 intersect sides $\overline{AB}$ and $\overline{AC}$ at D and E, respectively, as shown in the diagram. Compute the area of the shaded region.



19. Suppose we have four real numbers a, b, c, d such that a is nonzero, a, b, c form a geometric sequence, in that order, and b, c, d form an arithmetic sequence, in that order. Compute the smallest possible value of $\frac{d}{a}$. (A geometric sequence is one where every succeeding term can be written as the previous term multiplied by a constant, and an arithmetic sequence is one where every succeeeding term can be written as the previous term added to a constant.)

20. The game Boddle uses eight cards numbered $6, 11, 12, 14, 24, 47, 54$, and n, where $0 \leq n \leq 56$. An integer D is announced, and players try to obtain two cards, which are not necessarily distinct, such that one of their differences (positive or negative) is congruent to D modulo 57. For example, if $D = 27$, then the pair 24 and 54 would work because $24 - 54 \equiv 27 \bmod 57$. Compute n such that this task is always possible for all D.

21. On regular hexagon $GOBEAR$ with side length 2, bears are initially placed at G, B, A, forming an equilateral triangle. At time $t = 0$, all of them move clockwise along the sides of the hexagon at the same pace, stopping once they have each traveled 1 unit. What is the total area swept out by the triangle formed by the three bears during their journey?

22. Given a positive integer n, let $s(n)$ denote the sum of the digits of n. Compute the largest positive integer n such that $n = s(n)^2 + 2s(n) - 2$.

23. For real numbers B, M, and T, we have $B^2 + M^2 + T^2 = 2022$ and $B + M + T = 72$. Compute the sum of the minimum and maximum possible values of T.

24. Triangle $\triangle BMT$ has $BM = 4$, $BT = 6$, and $MT = 8$. Point A lies on line $\overleftrightarrow{BM}$ and point Y lies on line $\overleftrightarrow{BT}$ such that $\overline{AY}$ is parallel to $\overline{MT}$ and the center of the circle inscribed in triangle $\triangle BAY$ lies on $\overline{MT}$. Compute AY.

25. Bayus has eight slips of paper, which are labeled 1, 2, 4, 8, 16, 32, 64, and 128. Uniformly at random, he draws three slips with replacement; suppose the three slips he draws are labeled a, b, and c. What is the probability that Bayus can form a quadratic polynomial with coefficients a, b, and c, in some order, with 2 distinct real roots?