

**Time limit:** 60 minutes.

**Instructions:** This test contains 20 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No calculators.**

1. David is taking a 50-question test, and he needs to answer at least 70% of the questions correctly in order to pass the test. What is the minimum number of questions he must answer correctly in order to pass the test?
2. You decide to flip a coin some number of times, and record each of the results. You stop flipping the coin once you have recorded either 20 heads, or 16 tails. What is the maximum number of times that you could have flipped the coin?
3. The width of a rectangle is half of its length. Its area is 98 square meters. What is the length of the rectangle, in meters?
4. Carol is twice as old as her younger brother, and Carol's mother is 4 times as old as Carol is. The total age of all three of them is 55. How old is Carol's mother?
5. What is the sum of all two-digit multiples of 9?
6. The number 2016 is divisible by its last two digits, meaning that 2016 is divisible by 16. What is the smallest integer larger than 2016 that is also divisible by its last two digits?
7. Let  $Q$  and  $R$  both be squares whose perimeters add to 80. The area of  $Q$  to the area of  $R$  is in a ratio of 16 : 1. Find the side length of  $Q$ .
8. How many 8-digit positive integers have the property that the digits are strictly increasing from left to right? For instance, 12356789 is an example of such a number, while 12337889 is not.
9. During a game, Steve Korrry attempts 20 free throws, making 16 of them. How many more free throws does he have to attempt to finish the game with 84% accuracy, assuming he makes them all?
10. How many different ways are there to arrange the letters *MILKTEA* such that *TEA* is a contiguous substring?  
For reference, the term "contiguous substring" means that the letters *TEA* appear in that order, all next to one another. For example, *MITEALK* would be such a string, while *TMIELKA* would not be.
11. Suppose you roll two fair 20-sided dice. What is the probability that their sum is divisible by 10?
12. Suppose that two of the three sides of an acute triangle have lengths 20 and 16, respectively. How many possible integer values are there for the length of the third side?
13. Suppose that between Beijing and Shanghai, an airplane travels 500 miles per hour, while a train travels at 300 miles per hour. You must leave for the airport 2 hours before your flight, and must leave for the train station 30 minutes before your train. Suppose that the two methods of transportation will take the same amount of time in total. What is the distance, in miles, between the two cities?

14. How many nondegenerate triangles (triangles where the three vertices are not collinear) with integer side lengths have a perimeter of 16? Two triangles are considered distinct if they are not congruent.
15. John can drive 100 miles per hour on a paved road and 30 miles per hour on a gravel road. If it takes John 100 minutes to drive a road that is 100 miles long, what fraction of the time does John spend on the paved road?
16. Alice rolls one pair of 6-sided dice, and Bob rolls another pair of 6-sided dice. What is the probability that at least one of Alice's dice shows the same number as at least one of Bob's dice?
17. When  $20^{16}$  is divided by  $16^{20}$  and expressed in decimal form, what is the number of digits to the right of the decimal point? Trailing zeroes should not be included.
18. Suppose you have a  $20 \times 16$  bar of chocolate squares. You want to break the bar into smaller chunks, so that after some sequence of breaks, no piece has an area of more than 5. What is the minimum possible number of times that you must break the bar?  
  
For an example of how breaking the chocolate works, suppose we have a  $2 \times 2$  bar and wish to break it entirely into  $1 \times 1$  bars. We can break it once to get two  $2 \times 1$  bars. Then, we would have to break each of these individual bars in half in order to get all the bars to be size  $1 \times 1$ , and we end up using 3 breaks in total.
19. A class of 10 students decides to form two distinguishable committees, each with 3 students. In how many ways can they do this, if the two committees can have no more than one student in common?
20. You have been told that you are allowed to draw a convex polygon in the Cartesian plane, with the requirements that each of the vertices has integer coordinates whose values range from 0 to 10 inclusive, and that no pair of vertices can share the same  $x$  or  $y$  coordinate value (so for example, you could not use both  $(1, 2)$  and  $(1, 4)$  in your polygon, but  $(1, 2)$  and  $(2, 1)$  is fine). What is the largest possible area that your polygon can have?